B.Sc Part II (Honrs)

Important theorems of cosets.

Theorem:- If G be a group and if H be the subgroup of G, then prove that the number of left cosets of H in G is equal to the number of right cosets of H in G.

Proof:- Let $f(aH) = Ha^{-1} \forall a \in H$ —— (i)

If aH is a left coset, then obviously Ha^{-1} is a right coset.

In order to prove ~~that~~ the number of left cosets of H in G is equal to the number of right cosets of H in G we first show that f is one-one and onto.

f is one-one:

We have $f(ah) = f(bH)$

$\Rightarrow Ha^{-1} = Hb^{-1}$ (by (i))

$\Rightarrow a^{-1}(b^{-1})^{-1} \in H$ $[\because Ha = Hb \Rightarrow ab^{-1} \in H]$

$\Rightarrow a^{-1}b \in H \Rightarrow a^{-1}bH = H$ $[\because h \in H \Rightarrow hH = H]$

$\Rightarrow aa^{-1}bH = aH \Rightarrow bH = aH$

$\therefore f$ is one-one.

f is onto: Let Ha is right coset $a^{-1}H$ is a left coset.

And $f(a^{-1}H) = H(a^{-1})^{-1}$ —— by (i)

$= Ha.$

Hence each right Coset Ha is the f-image of the left coset $a^{-1}H$. So f is onto.

Hence if H is a subgroup of G, then there is one-one function between the set of left cosets of H in G and the set of right Cosets of H in G. Thus we coclude that if the number of distinct right cosets of H in G in finite, then it will also equal to the number of distinct let cosets of H in G.

Proved.

Theorem:- Prove that in a group, the order of a^{-1} is the same as that of a.

Proof:- let order of $a = n$ and order of $(a^{-1}) = m$

When $o(a) = n \Rightarrow a^n = e$

$\Rightarrow (a^n)^{-1} = e^{-1}$

$\Rightarrow (a^{-1})^n = e$ $\quad [\because e^{-1} = e]$

Since $o(a^{-1}) = m$ ——— (1)

$\therefore m \leq n$.

Also when $o(a^{-1}) = m$

$\Rightarrow (a^{-1})^m = e$ ——— (2)

$\Rightarrow n \leq m$,

By (1) and (2) $n = m$

So order of a^{-1} is the same as that of a.

Anjani Kumar Singh.

27.04.2021